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Rapid and accurate fatigue assessment by an efficient critical plane algorithm: application to a FSAE car rear upright

A. Chiocca[a,*], M. Sgamma[a], F. Frendo[a], F. Bucchi[a]

[a] *Department of Civil and Industrial Engineering, University of Pisa, Pisa, Italy*

Abstract

The topic of material fatigue is widely discussed and researched in both scientific and industrial communities. Fatigue damage remains a significant issue for both metallic and non-metallic components, leading to unforeseen failures of in-service parts. Critical plane methods are particularly recommended in case of multiaxial fatigue assessment and have gained relevance as they allow for the identification of the component's critical location and early crack propagation. However, the standard method for calculating critical plane factors is time-consuming, utilizing nested for/end loops and, for that, is mainly applied in a research context, or when critical regions are already known. In many cases, the critical area of a component cannot be identified due to complex geometries and loads or time constraints. This becomes particularly relevant after topological optimization of components and, more generally, in lightweight design. An efficient algorithm for critical plane factors evaluation have been recently proposed by the authors. The algorithm applies to all critical plane factors that require the maximization of a specific parameter based on stress and strain components or a combination of them. The methodology is based on tensor invariants and coordinates transformation law. This paper presents and validate the proposed methodology through an automotive case study: the new algorithm was tested on a rear upright of a FSAE car, having complex geometry, subjected to non-proportional loading conditions. The efficient algorithm showed a significant reduction in computation time compared to the (blind search-for) standard plane scanning method, without any loss in solution accuracy.

Keywords: critical plane; multiaxial fatigue; fatigue assessment; computational cost; algorithm efficiency; finite element analysis; upright; lightweight design

1. Introduction

The topic of material fatigue it is relevant in the scientific and industrial community (Cowles (1989); Kaldellis and Zafirakis (2012); Koyama et al. (2017); Xu et al. (2021); Chiocca et al. (2022); Wagener and Chiocca (2021); Chiocca et al. (2021b)). The majority of in-service failures of components are attributed to fatigue failure Bhaumik et al. (2008), which is a major design challenge due to the complexities of real-world applications such as variable amplitude, ran-

* Corresponding author. Tel.: +39-050-2218011.
E-mail address: andrea.chiocca@unipi.it

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domness, and multiaxiality (Kuncham et al. (2022); Sgamma et al. (2023)). Nowadays, the current trend in the use of innovative materials and in lightweight design in the automotive industry, with the aims of reducing the overall impact on the environment and human health, has enhanced the need of efficient and accurate design tools. The use of finite element analysis (FEA) has become a standard methodology to account for the complexities in component geometries and applying the correct load history (Frendo et al. (2020); Chiocca et al. (2019, 2021a); Meneghetti et al. (2022)). However, simulations can be time-consuming, particularly during the results post-processing phase, where several methods for damage calculation can be employed, such as the energy-based (Lazzarin and Berto (2005); Berto and Lazzarin (2009); Mroziński (2019); Varvani-Farahani et al. (2007)) and stress/strain-based approaches (Taylor et al. (2002); Radaj et al. (2006); Findley (1959); Socie (1987)). Strain-based methods are suitable for low-cycle-fatigue regime, while stress-based methods are more often employed in the high-cycle-fatigue regime. Energetic criteria are subdivided into strain-energy-based criteria for low-cycle-fatigue applications (Macha and Sonsino (1999)) and stress-energy-based criteria for high-cycle-fatigue applications.

One particular family of methods is that based on the concept of critical plane (CP). It is a local method, according to which a given damage factor has to be evaluated in every possible orientation, for any given location over the model, thus determining the point and the plane orientation that experiences the greatest value of the damage parameter (Reis et al. (2014); El-sayed et al. (2018); Zhu et al. (2018); Cruces et al. (2022)). This plane is called the critical plane and represents the material orientation over which the crack originates and initially propagates. However, the implementation of the CP method is usually time-consuming, especially for three-dimensional models with complex load history and geometry, as it requires scanning several planes in the three-dimensional space through the use of nested *for/end* loops. The iterative process is slowed down as quantities unnecessary for the definition of the damage parameter are sometimes evaluated on each rotated plane. The implementation process may have to be applied to as many nodes as the model contains, as defining the critical region *a priori* is not always possible. In this context, the use of optimization algorithms can be useful to enable a comprehensive analysis of the component.

In the present paper, a rapid and accurate procedure for the fatigue assessment, based on a closed form solution for the critical plane orientation recently presented by the authors (see Chiocca et al. (2023)), is applied to a formula SAE racing car rear upright; the component is subjected to non-proportional loading condition. For the sake of simplicity and with the intention of showing the method capability, the component was subjected to a load cycle consisting of two different car on-track conditions: right-turn with braking and right-turn with acceleration. The geometry and loads applied on the component do not allow for a prior assessment of the critical region, thus a global assessment of the entire component was deemed necessary. The applied loads and the component geometry resulted from previous dynamic analysis and topological optimization studies performed by the University of Pisa's formula SAE team.

2. Standard plane scanning method for critical plane factors evaluation

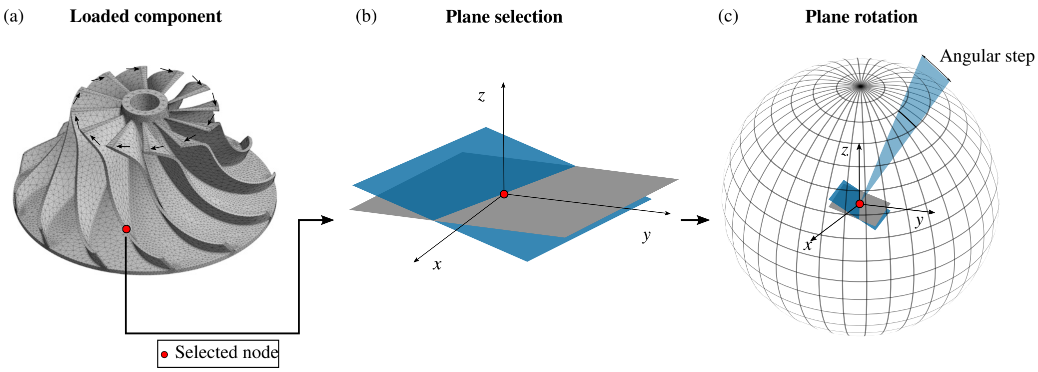

Fig. 1. (a) Generic finite element model of a loaded component, (b) rotated plane for the selected node, (c) spatial distribution of the unit vector's tip caused by the step-based rotation sequence.

The CP factor is an important parameter in predicting the fatigue life of a component. It is calculated using stresses and strains and represents the ability of the material to withstand fatigue under complex loading conditions. The

standard method of plane scanning is widely adopted in the literature for evaluating the CP factor.
The fatigue strength of a material is dependent on the time evolution of stress and strain tensors, which can be evaluated for every possible point in the component using nodes or integration points in finite element models (Figure 1a). The scanning plane method involves defining a plane and its unit normal vector, and incrementally rotating the plane by fixed angular steps along possible directions (i.e., normally done using two angles), as shown in Figure 1b. This allows for a precise evaluation of stresses and strains in all possible directions. The spatial distribution of the unit vector's tip caused by the step-based rotation sequence is shown in Figure 1c. This method is a "blind search-for" method, which results inherently inefficient as it requires scanning all possible planes before selecting the one where the CP factor is maximum. This is done through the use of nested *for/end* loops and requires long computation times for each individual node.

3. Efficient evaluation of critical plane factors

In the following, for the sake of clarity and without loosing generality, two critical plane factors will be considered as a reference:

- *Fatemi-Socie* critical plane factor (FS) Fatemi and Socie (1988)

$$FS = \frac{\Delta\gamma_{max}}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau'_f}{G}(2N_f)^{b_0} + \gamma'_f(2N_f)^{c_0} \tag{1}$$

  where k is the material parameter found by fitting the uniaxial experimental data against the pure torsion data, $\Delta\gamma_{max}$ is the maximum shear strain range, $\sigma_{n,max}$ is the maximum normal stress acting on the $\Delta\gamma_{max}$ plane and σ_y is the material yield strength. This critical plane model is typically used for materials that tend to shear cracking. The right-hand side of Equation 1 represents the shear strain-life curve for the material under consideration whose parameters are given in Table 1;
- *Smith-Watson-Topper* critical plane factor including Socie's modification (SWT) Socie (1987)

$$SWT = \frac{\Delta\varepsilon_1}{2}\sigma_{n,max} = \frac{\sigma'^2_f}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c} \tag{2}$$

  where $\frac{\Delta\varepsilon_1}{2}$ is the amplitude of the maximum principal strain and $\sigma_{n,max}$ is the maximum stress on the maximum principal strain plane. This CP model is typically employed for materials that tend to tensile cracking contrary to FS model. The right-hand side of Equation 2 represents the uniaxial strain-life curve for the material under consideration whose parameters are given in Table 1.

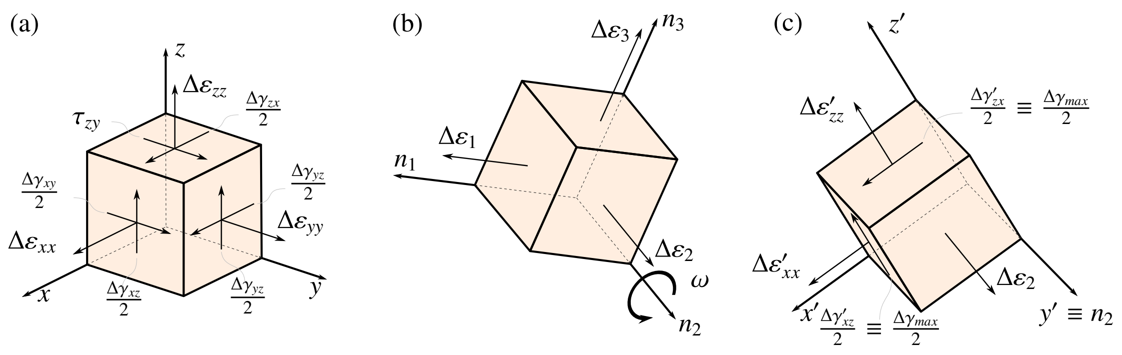

Fig. 2. Strain tensor range rotations represented through the infinitesimal material volume (Chiocca et al. (2023)): (a) strain range quantities for a generic orientation of the reference frame, (b) strain range components in the principal (for the strain range) reference frame and (c) strain range components after a $\frac{\pi}{4}$ rotation of the principal reference frame around n_2.

The two critical plane factors will be evaluated based on the results of a finite element analysis employing the closed form solution, which allows to speed up the CP calculation for each node, as already described by Chiocca et al. (2023). The efficient algorithm directly evaluate CP factors avoiding the spatial plane scanning, being based on the analytical solution obtained considering tensor invariants and coordinates transformation law. Figure 2 shows a graphical overview of the method. It is worth noting that the ω angle shown Figure 2 varies according to the CP method employed (see Equations 1 and 2), resulting in $\omega = \frac{\pi}{4}$ for FS and $\omega = 0$ for SWT.

4. Material and methods

The component investigated in the following is a FSAE car rear upright as shown in Figure 3. An upright is part of the vehicle wheel assembly that allows to transfer loads from the wheels to the suspension systems. The component was manufactured by CNC machining a single piece of 7075-T6 aluminum, whose properties are shown in Table 1–2. As it can be seen, the component has a complex geometry and there are many potentially critical notches, where fatigue cracks may nucleate.

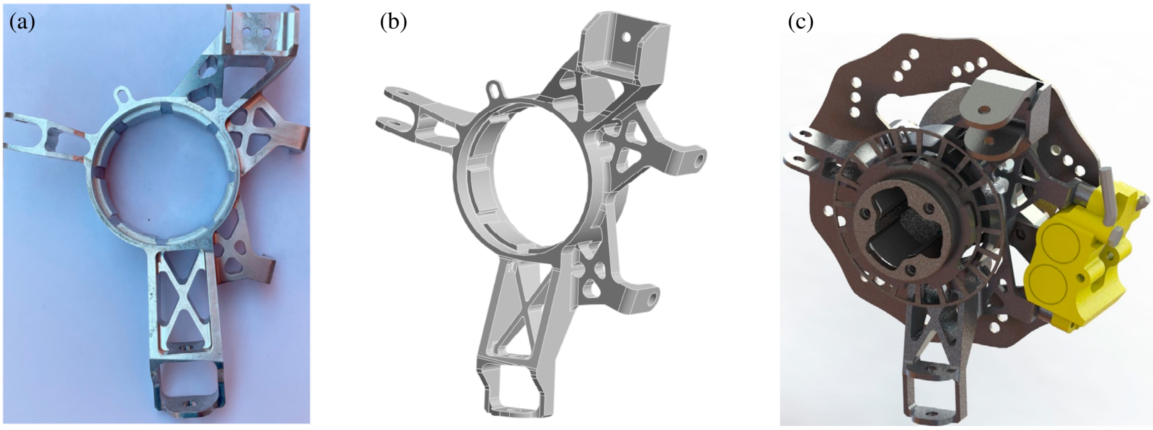

Fig. 3. Upright component used for the fatigue assessment analysis; (a) machined upright made of 7075-T6 aluminium, (b) its CAD model and (c) a sub-assembly of the upright and the brake system.

Table 1. Shear fatigue properties and uniaxial fatigue properties of 7075-T6 aluminum obtained from Gates and Fatemi (2017) (i.e., S.f. stands for "Shear fatigue" and F. stands for "Fatigue").

S.f. strength coefficient (τ'_f)	S.f. strength exponent (b_0)	S.f. ductility coefficient (γ'_f)	S.f. ductility exponent (c_0)
797 MPa	−0.126	5.42	−1.173
F. strength coefficient (σ'_f)	F. strength exponent (b)	F. ductility coefficient (ε'_f)	F. ductility exponent (c)
1235 MPa	−0.138	0.243	−0.710

Table 2. Monotonic properties and cyclic deformation properties of 7075-T6 aluminum obtained from Gates and Fatemi (2017).

0.2% Yield strength (σ_y)	Modulus of elasticity (E)	Elastic Poisson's ratio (ν)	Ultimate tensile strength (σ_u)
501 MPa	71.7 GPa	0.306	561 MPa

0.2% Cyclic axial yield strength (σ'_y)	Cyclic axial strength coefficient (K')	Cyclic axial hardening exponent (n')
518 MPa	845 GPa	0.079

The loading conditions used in the finite element model were obtained from a previous dynamic analysis performed by the SAE formula racing team of the University of Pisa during the 2020 season.
The CAD model of the component was used to perform a finite element analysis by means of Ansys Workbench software. The loading conditions, shown in Figure 4, are composed of two steps which are identified during a particularly severe corner. The first load condition (i.e., load step 1 of the analysis as shown in Figure 4a) refers to a right-turn with braking, while the second load condition (i.e., load step 2 of the analysis as shown in Figure 4b) refers to a right-turn with acceleration. Bearing remote displacement constraints A and B of Figure 4c were imposed on the component in order to represent the upright bearings, while a dummy remote displacement C was necessarily introduced to eliminate the system rotation lability around y-axis. During post-processing, the reaction on the dummy constraint was verified to be zero. The analysis was solved by imposing linear elastic and elastic-plastic material properties given in Table 2–2. A multilinear kinematic hardening law was employed based on the Ramberg-Osgood parameters of Table 2.

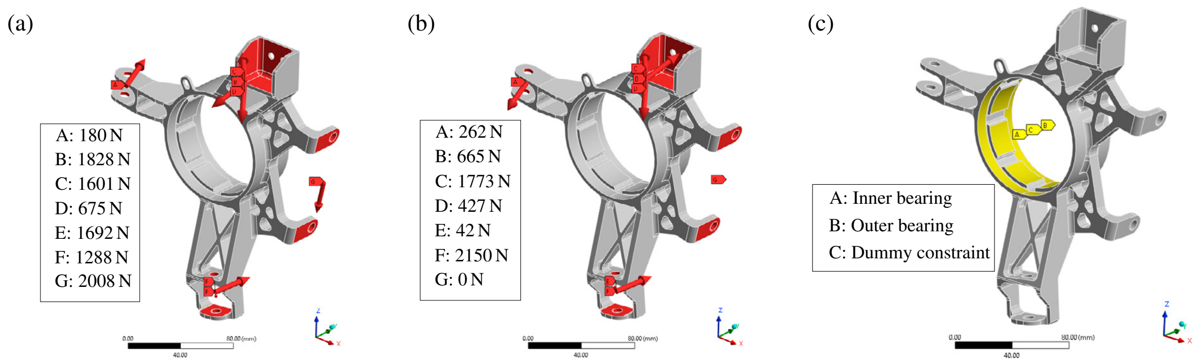

Fig. 4. Boundary conditions applied to the component: loadings are shown in red (a) for the first load step, (b) for the second load step and (c) the displacement constraints are shown in yellow.

5. Results and discussion

From the FE analysis, it is possible to extract all stress and strain data for each node and for each load step. This ensures the calculation of the critical plane factors presented in Equation 1–2. Given the component's geometric complexity and the non-proportional loading sequence applied, the identification of the critical region for fatigue crack nucleation prior to the calculation of the critical plane factors is very challenging . In this case, it is therefore necessary to proceed to an overall assessment of the damage factor for the entire component. However, when employing the plane scanning method, the damage factor calculation over the entire model (i.e. 61367 nodes) would require several hours (i.e. 7.3 h). Applying the optimized algorithm, on the other hand, brings the intended result way faster (i.e. 63 s, when the efficient code is directly implemented in Ansys Workbench) and thus greatly simplifies the fatigue assessment of the whole component.
In Figure 5, the upright color map is shown for both the FS and SWT CP factors evaluated by means of the efficient method and the standard scanning plane method. It is worth noting that, each CP method provide the damage obtained in a fatigue loading cycle (i.e., the two load steps in the finite element analysis) making the FEA results useful for fatigue assessment. In both cases, the component critical region is located in the low-arm of the upright where the load is transferred from the wheel to the tie rod and rear lower arm. Additionally, CP-life curves are provided for each method based on Equation 1–2. Both FS and SWT models provide comparable damage, with a minimum fatigue endurance of 1.8×10^4 cycles, which is greater than what is required for the given application.

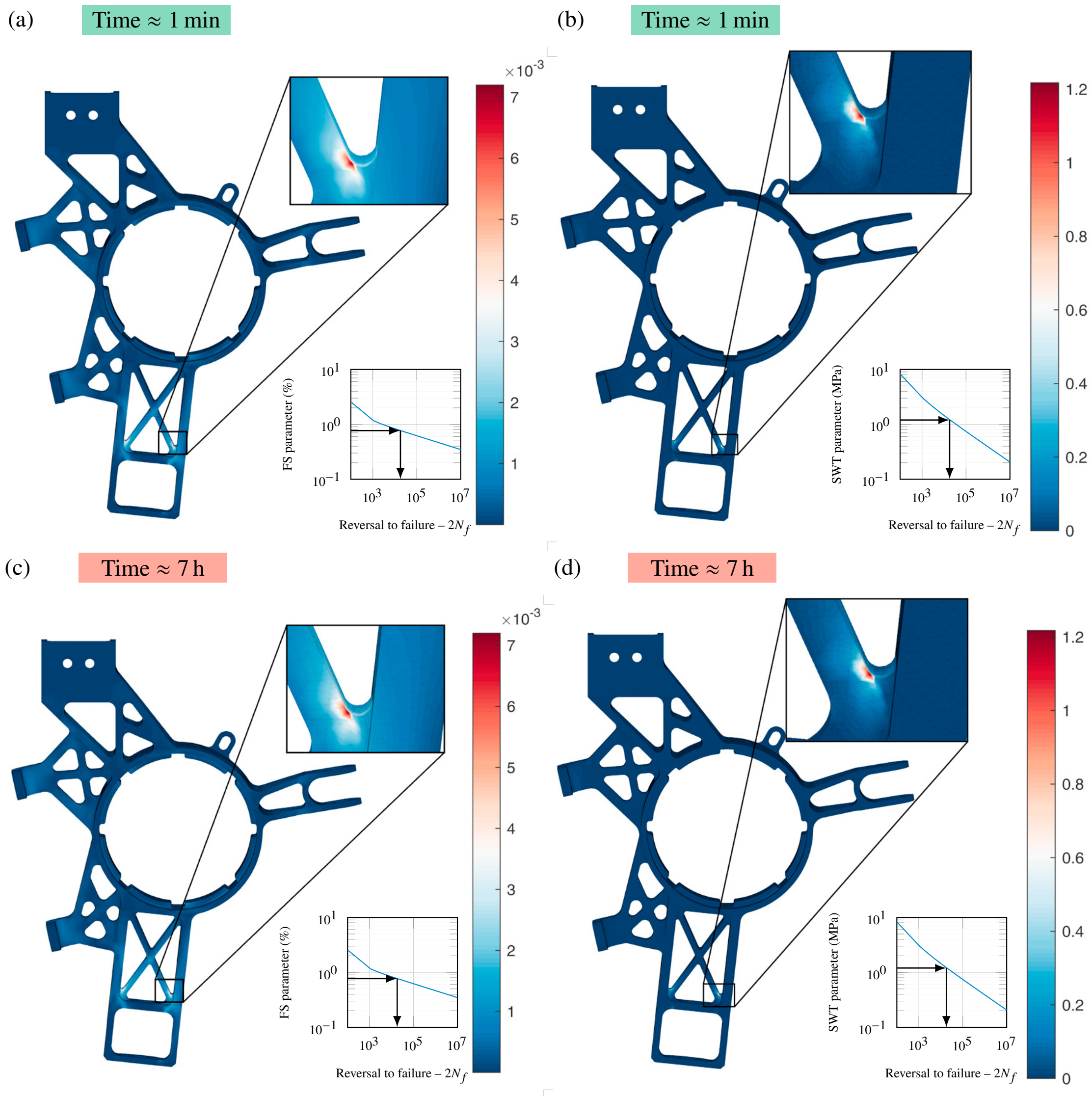

Fig. 5. Critical plane factors' colour map for: (a) *Fatemi-Socie* model and (b) *Smith-Watson-Topper* model solved by means of the efficient method and (c) *Fatemi-Socie* model and (d) *Smith-Watson-Topper* model solved by means of the scanning plane method.

6. Conclusions

In the paper a method for a rapid and accurate fatigue assessment of a component having complex geometry and subjected to non proportional loading was presented. The procedure is based on a closed form solution of the critical plane orientation and critical plane factor which has been recently published by the authors. An upright component belonging to a formula SAE race car from the University of Pisa's Formula Student team was selected with the aim of explain the method. The imposed loading and constraints were derived from previous dynamic analyses conducted within the Formula Student competition 2020. Due to the component's complexity and non-proportional loading condition, the damage on the entire component was investigated, as the critical region was hardly identifiable beforehand. The fatigue analysis was performed employing two critical plane methods namely *Fatemi-Socie* and *Smith-Watson-Topper* critical plane factors. The use of the efficient algorithm allowed for a massive reduction in the critical plane factor computation effort, decreasing the calculation time from about 7.3 h using the standard plane scanning method to 67 s, using the efficient method. It was shown that the accuracy of the result was not compromised, as the efficient method produces an analytically correct solution of the implemented critical plane factors. Efficient critical plane calculation algorithms open up new possibilities for the use of these methods in the industry, enabling rapid analysis of complex models subjected to non-proportional fatigue loading conditions. This becomes particularly attractive, also in case of complex geometries such as those obtained with topological optimization, with the intention of mass reduction.

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